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This report summarizes research activities carried out under the subject grant during the period May 1982 to April 1983. Specifically, a strength-of material-type fracture mechanics for beams has been development which leads, for example, to an exceedingly simple formula for the stress intensity factor of a cracked beam. Further, several aspects of conservation laws in elasticity have been clarified and extended to fluid dynamics, elucidating the classical Blasius Theorem. Moreover, conservation laws for dielectrics have been derived which will be used later to determine energy release rates. Finally, procedures to claculate upper and lower bounds of energy-release rates in elastic structural elements have been developed and will be used in the future to calculate stress intensity factors.

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PATH-INDEPENDENT INTEGRALS AND FRACTURE MECHANICS

at

Stanford University

Research sponsored by

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

submitted by

Professor Alicia G. Herrmann

Professor George Herrmann

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June 1983

INTRODUCTION

During the period covered by this report, significant advances have been made in establishing new conservation laws for certain systems, examining associated path-independent integrals and applying these relations to fracture and defect mechanics. These advances are briefly reviewed in the body of this report, making reference to published papers and to manuscripts under preparation. In order to place these research efforts into a proper perspective, it was deemed desirable to restate the general goals and aims as established in the original proposal.

THE GENERAL AIMS AND GOALS

A successful design of modern and complex aerospace structural systems and components will require increased shape control and dimensional stability. The former will place more stringent requirements on our ability to analyze wave propagation phenomena and to study thermal effects in structures, among others. The latter will involve increased demands on our skills to advance thermomechanical constitutive models and to perform coupled thermal structural analyses. In all of the above areas a further consideration has to be brought into awareness. The materials which are being used for structural applications contain flaws, such as microcracks and microvoids of various types and various origins. In addition, structural components may be fabricated from composite materials, thus creating numerous boundaries between different materials.

It can be shown that both defects and nonhomogeneities of various kinds can all be described mathematically with the aid of a quantity which may be called the material momentum tensor. This quantity, in a different form, was originally introduced into continuum mechanics by J. D. Eshelby, in 1951, which he called the energy-momentum tensor, using the same terminology as in electromagnetism, where

this quantity had been introduced a few decades earlier. It is believed that the term material momentum tensor is more accurate, since energy is related to invariance in time and thus does not occur in static problems. The term Eshelby tensor might also be appropriate, since Eshelby spent considerable effort in promoting the notion of this quantity.

Without attempting to give here a comprehensive account of the derivation and relevance of the material momentum tensor, it might suffice to mention that recent work by the authors of this report has placed the heretofore somewhat mysterious material momentum tensor into its proper, and hopefully better understood, perspective. In static problems it can be stated that the material momentum tensor plays precisely the same role in material space as the usual stress tensor in physical space. These quite distinct roles have been rather obscure in the past because researchers, including Eshelby, have used the displacement vector as one of the basic quantities which couples the material and physical space.

Further, the integrand of the by now well-known J-integral of fracture mechanics is strongly related to the material momentum tensor. It is recalled here, briefly, that in a plane static problem of elasticity, the J-integral is a line integral which, if taken along a closed contour, will give information regarding defects (such as cracks) and nonhomogeneities within the contour. If the contour encloses no defects or nonhomogeneities, the value of J will be zero. If the contour contains a single crack tip, the value of J will be identical to the crack extension force, or energy release rate. In this case, the J-integral is also said to be path-independent because whatever the contour surrounding the single crack tip, its value remains the same. This is somewhat reminiscent of the calculus of residues. In three dimensions, the J-integral is a three-dimensional vector defined as an integral over a closed surface.

The J-integral is one of several so-called conservation integrals which have been recently established. The various contributions which led to the present state-of-art were reviewed in the original proposal and this material will not be reproduced here. Suffice it to mention that while the J-integral is related to translational invariance, the other two conservation integrals are related to rotation and self-similar expansion, respectively.

The general objectives of our research, then, consist in improving our general capabilities of dealing with cracks and other defects in structural elements under a variety of circumstances. Specifically, we have been successful in reducing some problems of fracture mechanics to the level of strengh-of-material-type analysis; we have devised much simplified and more lucid methods of deducing the basic conservation laws; we have extended these conservation laws to coupled fields, such as dielectrics and thermoelasticity and we even have pointed out applications of conservation (or balance) laws in fluid dynamics.

WORK COMPLETED AND IN PROGRESS UNDER THE SUBJECT GRANT

A. Strength-of-materials-type Fracture Mechanics

One of the most important tasks of fracture mechanics is the calculation of crack extension forces (i.e., energy release rates), which via Irwin's relation determine the stress intensity factors. A large literature exists on this subject, formulating the problem in two or three dimensions on the basis of the theory of elasticity. Quite often extensive analytical and/or numerical work may be involved in achieving the desired goal.

It is a trite observation to make that as regards stress analysis of structural elements without defects (i.e., cracks), approximate theories of the strength-of-materials-type are often most successful, and there is no need to make recourse to the more inclusive theory of elasticity. For example, deflection of beams are calculated most commonly on the basis of the elementary Bernoulli-Euler theory.

The thought occured to us that it may be worthwhile to explore the possibility of establishing strength-of-materials-type theories of structural elements with cracks. It was natural to start with the elementary beam theory and to include the presence of cracks in such a manner as to be able to calculate crack extension forces. It appears that this is indeed possible and a surprisingly simple formula permits the calculation of stress intensity factors of cracked beams.

Consider a beam of nominal bending stiffness EI with a crack normal to the beam axis at a certain cross-section, such that the bending stiffness there is reduced to EI*. The results of our analysis indicate that the stress intensity factor K may be calculated by the formula

$$K = \frac{6M}{bh^{3/2}} \sqrt{\frac{1}{3}\left(\frac{I}{I^*} - 1\right)}$$

where M is the bending moment at the cracked cross-section; b is the depth and h the height of the beam, I is the moment of inertia of the uncracked and I*, the moment of inertia of the cracked cross-section. This simple formula has been applied to beams with symmetrical central and edge cracks and the results compared with those available in the literature. Astonishingly good agreement has been obtained over the full range of crack depths. A technical report summarizing this work is in process of preparation. In addition, it is planned to investigate other approximate theories of structural elements with cracks, such as, for example, theories of torsion and Timoshenko-type theories of beams.

B. On Conservation Laws in Elasticity

It is recalled that three independent material conservation laws exist in the theory of elastic solids which have been labeled as J, L, and M integrals. It has been observed that even though the L integral is related to the energy release rate due to a virtual rotation, it has not been expressed as the cross product of a position vector and a force. This is somewhat surprising since one would expect the integrand of the L integral to be expressible as a moment of the material force which is the integrand of the J integral.

In ref. [1] we have shown that in a complete nonlinear formulation, the corresponding conservation law has the form of the divergence of the cross product of the material coordinate and the material force.

It has been shown in a later paper, ref. [2], that actually both forms are equivalent in linearized elastostatics. It appears to us, however, that our form is more general, offering the possibility of extensions to dynamics and to non-linear behavior. To support this contention, a further study has been undertaken in order to clarify the matter some more with a view of treating nonlinear coupled fiels such as dielectrics and thermoelasticity.

In ref. [3], we have shown in a very simple manner how the three different conservation laws (originally proposed by Günther and independently by Knowles and Sternberg) are intimately and inseparably related to the postulated quadratic form of the strain energy. For the L integral, isotropy must also be assumed. Thus, a new general approach similar to that employed in ref. [1] should be used and only later should various approximations of the already derived conservation laws be introduced.

The method used in Ref. [3] is very simple: We consider the derivative of the strain energy W to derive the J integral and the moment $X_i W$, (X_i is the material coordinate) to obtain the L and M integrals. To obtain the L integral we apply the invariant curl operator, while for the M integral the divergence operator is employed. The results obtained are, of course, valid for both cartesian and curvi-linear coordinate systems. The property of the M integral being rather different in two and three dimensions becomes now most transparent.

In still another paper, ref. [4], we have shown that the path-independent integrals, which until now were applied only to fracture mechanics in solids, have some counterparts in fluid dynamics. The well-known Blasius Theorem giving the resultant force and the resultant moment acting on a solid in an incompressible

fluid in steady flow is used as an example and the correspondence of the resultant force and the resultant moment to the J and L integral for a defect in an elastic solid are pointed out. The L integral in this case has the form of a cross product as derived in ref. [1].

A further paper on some other aspects of the Blasius Theorem, which have been overlooked in the past, is currently under preparation.

C. Conservation Laws for Dielectrics

It is observed that polarizable solids deform when subjected to an electric field. If such a material contains a defect, i.e., a crack, the external field not only produces additional stresses, but it may have an influence on crack propagation. To study the effects of the additional field, the path-independent integrals that can be useful in fractue mechanics are sought for a dielectric material interacting with the external electric field. In elastostatics the path-independent integrals, namely J, L, and M, when evaluated along a path enclosing a defect give values which are related to energy release rates. They can be interpreted as the material force (J), Moment (L), and self-similar expansion moment (M) acting on the elastic singularity or inhomogeneity. In the absence of the defect, these integrals become conservation laws for the elastic material. The fact that the conservation laws and associated path-independent integrals are physically meaningful and useful in elastic fracture mechanics, provides the motivation to derive conservation laws for dielectrics.

Such conservation laws have now been established. Progress is being currently made to relate these new conservation laws for the coupled field to energy release rates such that they can be used to characterize a crack which is subjected to both mechanical loading and an external electric field. The boundary value problem for such a crack in an elastic sheet has to be solved to find the stress and the electric field distributions. It will be of interest to determine whether an applied electric field will enhance or inhibit crack growth.

D. Thermoelasticity

For coupled thermoelasticity a Lagrangian function has been constructed, Ref. [5], such that not only the well-known equations of thermoelasticity and heat conduction can be derived but also material conservation laws. These conservation laws reduce to the well-known J, L, and M integrals for isothermal deformations. It turns out that in thermoelasticity, due to the dissipative character of the system, the material momentum depends on a time interval rather than on an instant of time. The balance of material momentum can be integrated over time to produce a relation reminiscent of the impulse-momentum equation in classical mechanics.

Considerable difficulties of conceptual nature have been encountered in relating the new balance laws to energy release rates. These difficulties have been overcome recently and current efforts are directed to problems of thermoelasticity generated by steady heat flow in plates containing cracks and voids of various shapes.

E. Bounds for Energy-release Rates

The problem of bounds in numerical calculations of energy release-rates in stressed elastic bodies is of great interest in fracture and defect mechanics. Earlier work by another investigator was based on bounds obtained for the potential energies before and after crack growth. It turned out, however, that the bounds on energies are not of great value if one is interested in calculating bounds for energy release-rates.

Based on some recently developed extremum principles for bodies subjected to a prescribed change of material properties, energy release increments for non-homogeneous elastic bodies have been calculated. As a particular example, upper and lower bounds for the energy change of a rectangular elastic speciman with an

edge crack subjected to uniform tension in mode I, Ref. [6], have been calculated. Even though the kinematically admissible functions were very simple, each containing only one free perameter, nontrivial bounds have been obtained. The newly established method holds great promise for future numerical calculations.

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6. Pawel Rafalski and Alicia Golebiewska-Herrmann, "Upper and Lower Bounds for the Energy-release Rates in an Elastic Body," To be published in Int. J. Solids, Structures.

PERSONNEL

In addition to the principal investigators, the following graduate students (PH. D. Candidates) were associated with this project.

G. Francfort (Degree awarded in June 1982)

E. Pak

INTERACTIONS (Coupling Activities)

1. Conference on Nonlinear Behavior of Fluids NBS, Boulder, Colorado, June 1982. Invited lecture entitled "On the Lagrangian Formulation of Continuum Mechanics," presented by A. G. Herrmann.

2. Special all-day workshop for research staff of General Electric Company at Schenectady, New York, organized for G. Herrmann, June 1982.

3. Ninth National Congress of Applied Mechanics, Cornell University, June 1982. Paper entitled "A Material Momentum for Thermoelasticity," given by A.G. Herrmann.
4. Special discussions on static and dynamic fracture of structural elements at the Solid Mechanics Institute in Freiburg, Germany, A. G. Herrmann and G. Herrmann, July 1982.
5. Invited seminar lecture in the Applied Mechanics series at Stanford University on "Fundamentals of Fracture Mechanics," presented by G. Herrmann, October 1982.
6. ASME Winter Annual Meeting, Phoenix, Arizona. Invited paper entitled "Conservation Laws and Material Momentum for Thermoelasticity," (co-author Gilles Francfort), presented by A. G. Herrmann, Nov. 1982.
7. Second Cairo University Conference on Mechanical Design and Production, "Characterization of Cracks and Other Defects in Elastic and Thermoelastic Solids," general lecture by A. G. Herrmann, Dec. 1982.
8. Conference on Mechanics of Solid Continua, organized by Prof. H. Lippmann (Technical University of Munich) and G. Herrmann at the Mathematical Research Institute in Oberwolfach, Germany, Jan. 1983. Invited paper "A New Variational Principle and Conservation Laws for Thermoelasticity," presented by A. G. Herrmann.
9. Special two-day workshop on Integral Criteria in Fracture, organized for A. G. Herrmann and G. Herrmann at the Technical University of Darmstadt, Germany with participants from several European countries, January 1983. General lecture by A. G. Herrmann on "From Conservation Laws to Path-Independent Integrals." General lecture by G. Herrmann on "The Mechanics of a Plane Crack."
10. Invited lecture on crack mechanics by G. Herrmann at the Technical University of Munich, Jan. 1983.
11. Participation by G. Herrmann in a workshop on Dynamic Fracture at California Institute of Technology, Feb. 1983.